INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI
DEPARTMENT OF MATHEMATICS AND STATISTICS

MA6024-PARTIAL DIFFERENTIAL EQUATIONS

Assignment-1 [100 Marks]

1. Classify the following PDEs as linear, nonlinear, semilinear, quasilinear, homogeneous and inhomogeneous, provide reason [$5 \times 3 = 15$]

   (a) $u_t - u_{xx} + 1 = 0$

   (b) $u_t - uxx + xu = 0$

   (c) $u_t + u_{xxxx} + \sqrt{1+u} = 0$

   (d) $u_{tt} - u_{xx} + x^2 = 0$

   (e) $u_x + 3u = e^x$

2. Find the general solution of the following PDEs. $u = u(x, y, z)$ [$4 \times 5 = 20$]

   (a) $u_{xx} = 12xy$

   (b) $u_{xy} + u_x = 0$

   (c) $u_x + 2u = y$

   (d) $u_x - zu = y - z$

3. Find all solutions $u(x, y)$ of the PDE $u_x - 2u = 0$ which also satisfy the additional requirement that [$3 \times 5 = 15$]

   (a) $u(0, y) = y^2$

   (b) $u(1, y) = y^2$

   (c) $u(x, 1) = x^2$

4. Separate the PDE into a system of ODEs [$3 \times 5 = 15$]

   (a) $u_{xx} + u_y + u = 0$

   (b) $y_x^u + x^2 u_y = 0$

   (c) $u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0$

5. Show that the characteristics of $u_t + 5uu_x = 0$ with $u(x, 0) = f(x)$ are straight lines. [10]

6. Solve $u_x - 4u_y + u = 0$ using method of characteristics. [5]

7. Solve $5u_x - 3u_y + u_z = 0, u(1, y, z) = y \cos z$ using method of characteristics. [10]

8. Solve the equation $yu_x + xu_y = 0$ with $u(0, y) = e^{-y^2}$. In which region of the xy plane is the solution uniquely determined? [10]

INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI
DEPARTMENT OF MATHEMATICS AND STATISTICS

MA6024-PARTIAL DIFFERENTIAL EQUATIONS

Assignment-1 [100 Marks]

1. Classify the following PDEs as linear, nonlinear, semilinear, quasilinear, homogeneous and inhomogeneous, provide reason [$5 \times 3 = 15$]

   (a) $xu_x^2 - yu_y^2 = 2$

   (b) $xu_x - yu_y = xu^2$

   (c) $\sin(1 + u_{xx})^2 + u^3 = \sin x$

   (d) $u_{tt} - u_{xx} + u_{yy} = \sin u$

   (e) $\frac{1}{2^k} \sum_{|\alpha|=5} D^\alpha u = \sin u$

2. Consider the convection equation $u_t + cu_x = 0$ where c is constant. [$4 \times 5 = 20$]

   (a) Show that $u = \sin(x - ct), u = \cos(x - ct)$ and $u = 5(x - ct)^2$ are solutions

   (b) Show that $u = 7\sin(x - ct), u = 3\cos(x - ct)$ and $u = 7\sin(x - ct) - 3\cos(x - ct)$ are solutions

   (c) Show that $u = f(x - ct)$ is a solution for any function f.

3. Find all solutions $u(x, y)$ of the PDE $u_y = 2x$ which also satisfy the additional requirement that [$3 \times 5 = 15$]

   (a) $u(x, 0) = \sin x$

   (b) $u(x, 3) = \sin x$

   (c) $u(0, y) = 3y$

4. Separate the PDE into a system of ODEs [$3 \times 5 = 15$]

   (a) $5u_x + 4u_y - 2u = 0$

   (b) $u_{xx} - u_y + u = 0$

   (c) $x^2u_{xx} + 2u_x - 3u_y - yu = 0$

5. Let ϕ be a smooth function with compact support. Consider the initial value problem for the damped Burger's equation [$2 \times 5 = 10$]

$$u_t + uu_x + u = 0, \quad -\infty < x < \infty, t > 0$$

$$u(x, 0) = \phi(x)$$

   (a) Find an implicit equation for the classical solution

   (b) Show that $|u_x|$ is bounded if $\phi'(x) \geq -1$. That is, show that $|u_x| \nrightarrow \infty$ in finite time if $\phi'(x) \geq -1$.

6. Solve $(1+x^2)u_x+u_y = 0$ using method of characteristics. Sketch some of the characteristic curves [5]

7. Solve the equation $u_x + u_y + u = e^{x+2y}$ with $u(x, 0) = 0$. [10]

8. Solve the equation $\sqrt{1 - x^2}u_x + u_y$ with $u(0, y) = y$. [10]

भारतीय प्रौद्योगिकी संस्थान तिरुपति

TIRUPATI

INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI
DEPARTMENT OF MATHEMATICS AND STATISTICS

MA6024-PARTIAL DIFFERENTIAL EQUATIONS

Assignment-1 [100 Marks]

---

1. Classify the following PDEs as linear, nonlinear, semilinear, quasilinear, homogeneous and inhomogeneous, provide reason [$5 \times 3 = 15$]

   (a) $u_{xx} + u_{yy} = |\nabla u|^2 u$

   (b) $u_t + u_{xxx} + uu_x = 0$

   (c) $u_x + \sqrt{1 - u_y^2} = 0, |u_y| \leq 1$

   (d) $u_x - (x + y)u_y - u^3 xy = e^x$

   (e) $\sum_{k=0}^{\infty} \frac{1}{2^k} \sum_{|\alpha|=5} a_\alpha(x, u) D^\alpha u = \sin \|x\|$

2. Find general solution of the following PDEs [$3 \times 5 = 15$]

   (a) $u_{xy} = y + 2x$

   (b) $u_y + xu = 2xy$

   (c) $4u_x + 5u_y - u = 0$

3. Find all solutions $u(x, y)$ of the PDE $u_x + 2u = 2y$ which also satisfy the additional requirement that [$3 \times 5 = 15$]

   (a) $u(0, y) = y$

   (b) $u(1, 3) = -0.5y$

   (c) $u(x, 0) = e^x$

4. Separate the PDE into a system of ODEs [$3 \times 5 = 15$]

   (a) $(t^2 + 1)u_{ttt} - 25e^x u_{xx} = 0$

   (b) $\sin(3x)u_{ttt} - 25(t^3 + 3t)u_{xx} = 0$

   (c) $x^2 u_{xx} + 2u_x - 3u_y - yu = 0$

5. Find the solution of the differential equation

$$\left(1 - \frac{m}{r}u\right) u_x - mMu_y = 0$$

   with

$$u(0, y) = \frac{My}{\rho - y}$$

   where m, r, ρ, M are constants, in a neighbourhood of $(0, 0)$. [10]

6. Solve $(1+x^2)u_x+u_y = 0$ using method of characteristics. Sketch some of the characteristic curves. [10]

7. Solve the equation $uu_x + u_y = 0, y > 0, -\infty < x < \infty$ with Cauchy data

$$u(x,0) = \begin{cases} \alpha^2 - x^2 & \text{if } |x| \leq \alpha \\ 0 & \text{if } |x| > \alpha \end{cases}$$

[10]

8. Solve the equation $(2x - y)y^2u_x + 8(y - 2x)x^2u_y = 2(4x^2 + y^2)u$ with $u(x,0) = \frac{1}{2x}$. [10]

INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI
DEPARTMENT OF MATHEMATICS AND STATISTICS

MA6024-PARTIAL DIFFERENTIAL EQUATIONS

Assignment-1 [100 Marks]

1. Classify the following PDEs as linear, nonlinear, semilinear, quasilinear, homogeneous and inhomogeneous, provide reason [$5 \times 3 = 15$]
   (a) $u_{xx} + u_t = 2u(1-u)$
   (b) $u_{xx} + 2u_{yy} - uu_x + 3u_y = 0$
   (c) $y^2 u_x + x^2 u_y = 0$
   (d) $u_{tt} - u_{xx} + u_{yy} + u_{zz} = \sin u$
   (e) $\nabla^2 u + e^u = 0$
2. Solve the equation $2u_t + 3u_x = 0, x \in \mathbb{R}, t > 0$ for the following Cauchy data.[$3 \times 5 = 15$]
   (a) $u(x,0) = \sin x$
   (b) $u(x,0) = x^3$
   (c) $u(x,0) = \cos x + x^2$
3. Find PDEs whose general solutions are of the form. [$3 \times 5 = 15$]
   (a) $u(x,y) = \phi(x+y) + \psi(x-2y)$
   (b) $u(x,y) = x\phi(x+y) + y\psi(x+y)$
   (c) $u(x,y) = \phi(xy) + \psi(x/y)$
4. Separate the PDE into a system of ODEs. [$3 \times 5 = 15$]
   (a) $u_{xx} - xu_y + xu = 0$
   (b) $-iu_t = u_{xx} - x^u$, here is $i^2 = -1$
   (c) $x^2 u_{xx} + 2u_x - 3u_y - yu = 0$
5. Find all solution of the equation $u_x + u_y = 0$ of the form $u(x,y) = X(x) + Y(y)$. [5]
6. Fin all possible solutions of the PDE $yu_x - xu_y = 0, u(x,2x) = x^4$. Sketch the initial curve and some of the characteristic curves [10]

7. Solve the equation $uu_x + u_t = 0, t > 0, -\infty < x < \infty$ with Cauchy data

$$u(x,0) = -\tanh\left(\frac{x}{\epsilon}\right), x \in \mathbb{R}$$

[15]

8. Show that any solution of the problem $u_t + uu_x = u, x \in \mathbb{R}, t > 0$ with Cauchy data $u(x,0) = u_0(x), x \in \mathbb{R}$ satisfies the following relation. [10]

$$u = e^t u_0(x - u + ue^{-t}), n = 1$$