Assignment – 1: Deadline: 20 November 2020

Indian Institute of Technology Tirupati

Department of Mathematics and Statistics

MA5023 – Differential Equations for Engineers – Assignment - 1

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Mathematical Modelling

Problem 1: If the temperature of a cake is 150°C when it leaves the oven and is 100°C ten minutes later, then when will it reach the room temperature 20°C?

Problem 2: A cylindrical water tank with height 2.25m and radius 2m has a 1cm hole at the bottom. The hole was blocked until the water level reaches its maximum height in the tank. After the tank reached its maximum capacity at 9 AM, the hole was released. What is the least time, can you find an empty tank after 9 AM?

Problem 3: (a) It was found that the hormone level of a patient varies w.r.to time. The rate of change of the hormone w.r.to time is the difference between the sinusoidal input of 24 hours from the thyroid gland and a continuous removal rate proportional to the level. Find and solve the hormone level model?

(b) In a room containing $20 \times 10^3\ m^3$ of air, 600 m^3 of fresh air flows in per minute, and the mixture (made practically uniform by circulating fans) is exhausted at a rate of 600 cubic metres per minute. What is the amount of fresh air at any time if there is no initial fresh air? After what time will 90% of the air be fresh?

Problem 4: Mixing problems occur quite frequently in the chemical industry. We explain here how to solve the basic model involving a single tank. The tank contains 1000 litres of water in which initially 100 kg of salt is dissolved. Brine runs in at a rate of 10-litre per min, and each litre contains 5 kg of dissolved salt. The mixture in the tank is kept uniform by stirring. Brine runs out at 10 litres per min. Find the amount of salt in the tank at any time *t*.

Problem 5: A model for the spread of contagious diseases is obtained by assuming that the rate of spread is proportional to the number of contacts between infected and non-infected persons, who are assumed to move freely among each other. $y' = k(1-y)y$

Problem 6: Suppose that the population of a certain kind of fish is given by the logistic equation

$$y' = (A - By)y$$

and fish are caught at a rate *Hy* proportional to *y*. Find the model
Schaefer Model:

$$y' = (A - H - By)y$$

Problem 7: On what interval does each of the following initial value problems have a unique solution?

$$y' = (\cot x)y + \tan x\,, y(2) = 3$$

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Problem 8: Experiments show that at each instant a radioactive substance decomposes and is thus decaying in time-proportional to the amount of substance present. Model it and solve it.

Problem 9: Determine whether a cylinder of radius 4 cm height 10 cm and weight 15 kg can float in a deep pool of water of weight density 62.5 $\frac{kg}{cm^3}$

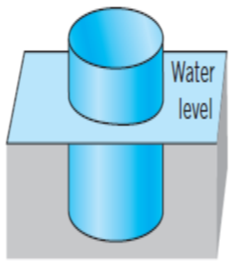
Problem 10: Solve the following IVP

(a) $xy' = y + 3x^4 \cos^2\left(\frac{y}{x}\right), y(1) = 0$

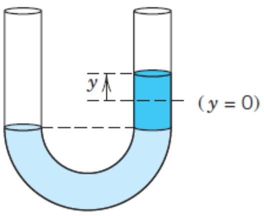
(b) $xy' + 4y = 8x^4, y(1) = 2$

(c) $(2xydx + dy)e^{x^2} = 0, y(0) = 2$

Problem 11: (a) According to Archimedean principle buoyance force equals the weight of the water displaced by the body. A cylindrical buoy of diameter 60cm is floating in water with its axis vertical. When depressed downward in the water and released, it vibrates with period 2 sec. What is its weight?



$$y" + \omega^2 y = 0$$

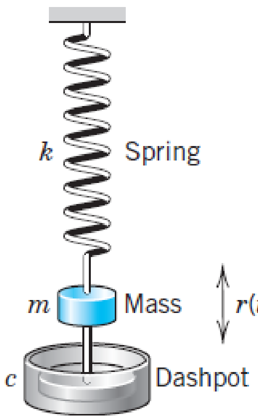


(b) What is the frequency of vibration of 5 litres of water in a U-Shaped tube of diameter 4cm? (Neglect friction).

$$y" + \omega^2 y = 0$$

(i) $m = 1, k = 3$ (ii) $m = 9, k = -1$

Hint: $\omega^2 = \frac{\pi r^2 k}{m}$



Problem 12: An ordinary coil spring resists extension as well as compression. We attach an iron mass at its lower end. When the system is at rest after attaching the iron ball, we say it as the initial position. When we pull the ball down, the system experiences a force. Further, we add a damping force to the system and an additional external force. What will be the mass-spring system?

External Force:

$$my" + cy' + ky = r(t)$$

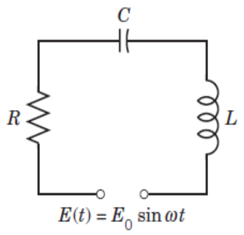
(a) $2y'' + 4y' + 6.5y = \cos(1.5t)$

(b) $y'' + 16y = 56 \cos t$

Problem 13: Let P be a particle of mass m, acted upon by an elastic force of attraction $F = -ky$, where y is the position vector, k is the coefficient of elasticity. Describe the motion of the particle.

Problem 14: Shock Absorber. What is the smallest value of the damping constant of a shock absorber in the suspension of a wheel of a car (consisting of a spring and an absorber) that will provide (theoretically) an oscillation free ride if the mass of the car is $2000\ kg$ and the spring constant is $4500\,\frac{kg}{s^2}$

Problem 15: Model the following RLC-Circuit and solve for I



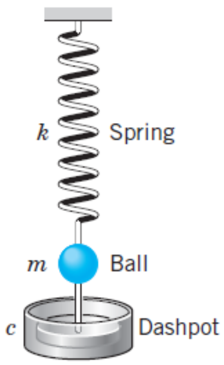
$$LI' + RI + \frac{I}{C} = E(t)$$

$$I = Q' \Rightarrow LQ'' + RQ' + \frac{Q}{C} = E(t)$$

Upon differentiation I w.r.to t, $LI'' + RI' + \frac{I}{C} = E'(t)$

(a) $R = 11\ \Omega, L = 0.1\ H, C = 10^{-2}F, E = 110\cos 50\ t\ \ V$

(b) $R = 8\ \Omega, L = 0.2\ H, C = 12.5 \times 10^{-3}F, E = 100\sin 10\ t\ V$



Problem 16: An ordinary coil spring resists extension as well as compression. We attach an iron ball at its lower end. When the system is at rest after attaching the iron ball, we saw it as the initial position. When we pull the ball down, the system experiences a force. Further, we add a damping force to the system. What will be the damped mass-spring system?

Damped Force

$$F_2 = -cy'$$

$$my'' + cy' + ky = 0$$

(a) $m = 1, k = 4.8, c = 5.76$
(b) $m = 1, k = 3, c = 2.5$

Problem 17: Solve the following

(a) $y'' - 2y' - 3y = 0, y(-1) = e, y(-1) = -e/4$
(b) $x^2y'' = 2y$
(c) $5x^2y'' + 23xy' + 16.2y = 0$
(d) $x^2y'' + 2xy' - 6y = 0, y(1) = 0.5, y'(1) = 1.5$

Problem 18: Find a second-order homogeneous linear ODE for which the given functions are solutions. Show linear independence by Wronskian. Solve the IVP

(a) $x^2, x^2\ln x\,, y(1) = 4, y'(1) = 6$
(b) $1, e^{3x}, y(0) = 2, y'(0) = -1$

Problem 19: Solve the following non-homogeneous linear ODE

(a) $y'' + y' + y = 2x\sin x$
(b) $y'' + 6y' + 9y = e^{-x}\cos 2x\,, y(0) = 1, y'(0) = -1$
(c) $y'' - 4y' + 4y = 5t^3 - 2 - t^2e^{2t} + 4e^{2t}\cos t$

Problem 20: Solve by using the variation of Parameters

(a) $x^2y'' + xy' - 9y = 48x^5$