भारतीय प्रौद्योगिकी संस्थान तिरुपति

TIRUPATI

Assignment – 2 Deadline: 14 December 2020

Indian Institute of Technology Tirupati

Department of Mathematics and Statistics

MA5023 – Differential Equations for Engineers – Assignment - 2

Green's function and Power Series

Problem 1: Find the particular solution of the following ODE using **Green's function**

a) $y'' + 4y = \sin 2x\,, y(0) = 1, y'(0) = -2$ (b) $y'' + 4y = 3, y'(0) = 0, y\left(\frac{\pi}{2}\right) = 0$

(c) $y'' + 4y = f(x), y(0) = 1, y'(0) = -2, f(x) = \begin{cases} 0 & x < 0 \\ \sin 2x & 0 \le x \le 2\pi \\ 0 & x > 2\pi \end{cases}$

Problem 2: Find the radius and interval of convergence of the following **power series**

$$(a) \sum_{m=0}^{\infty} \frac{(-1)^{n+1}}{n} x^n \quad (b) \sum_{m=0}^{\infty} \frac{5^n}{n!} x^n$$

Problem 3: Find the power series solution for the following ODEs

a) $(1 + x)y' = y$ (b) $y' = -2xy$

Problem 4: Identity ordinary, singular, regular singular and irregular singular points for the following ODEs.

a) $x^3y'' + 4x^2y' + 3y = 0$ (b) $(x^2 + x - 6)y'' + (x + 3)y' + (x - 2)y = 0$

Problem 5: Use the **Frobenius method** to find two independent solutions for the following ODE

a) $(x + 2)^2y'' + (x + 2)y' - y = 0$ (b) $x^2y'' + 6xy' + (4x^2 + 6)y = 0$

Legendre's and Bessel's equation

Problem 1: Legendre's equation

a) Using the following recurrence relation, list the first six Legendre polynomials
$$(k + 1)P_{k+1}(x) - (2k + 1)xP_k(x) + kP_{k-1}(x) = 0, k = 1,2,3$$
b) Show that the following DE can be transformed to Legendre's equation
$$sin\theta \frac{d^2y}{d\theta^2} + cos\theta \frac{dy}{d\theta} + n(n + 1)sin\theta y = 0$$

Problem 2: Bessel's equation: Prove the following

a) $J_0'(x) = -J_1(x), J_1'(x) = J_0(x) - J_1(x), J_2'(x) = \frac{1}{2}[J_1(x) - J_3(x)]$

b) $xJ_\nu'(x) = \nu J_\nu(x) - xJ_{\nu+1}(x)$

Problem 3: Find the general solution in terms of J_ν, Y_ν

a) $xy'' + 5y' + xy = 0, use\ y = \frac{u}{x^2}$

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Fourier Series and Fourier Integral

Problem 1: Find the Fourier coefficients of the periodic function $f(x)$

a) $f(x) = \begin{cases} -\cos^2 x & if \quad -\pi < x < 0 \\ \cos^2 x & if \quad 0 < x < \pi \end{cases}$ $and\ f(x + 2\pi) = f(x)$

b) $f(x) = |x|, -\pi < x < \pi$ $e)\ f(x) = x^2, -\pi < x < \pi$

Problem 2: Find the Fourier sine integral representation of the function

$$f(x) = \begin{cases} \frac{\pi}{2}\cos x & if \quad 0 < |x| < \frac{\pi}{2} \\ 0 & if \quad |x| \geq \frac{\pi}{2} \end{cases}$$

Problem 3: Find the Fourier cosine integral representation of the function

$$a)\ f(x) = \begin{cases} x^2 & if \quad 0 < x < 1 \\ 0 & if \quad x > 1 \end{cases} \ and\ b)\ f(x) = \begin{cases} \sin x & if \quad 0 < x < \pi \\ 0 & if \quad x > \pi \end{cases}$$

Problem 4: Find the Fourier sine integral representation of the function

$$a)\ f(x) = \begin{cases} x & if \quad 0 < x < 1 \\ 0 & if \quad x > 1 \end{cases} \text{ and } b)\ f(x) = \begin{cases} \cos x & if \quad 0 < x < \pi \\ 0 & if \quad x > \pi \end{cases}$$

Partial Differential Equations

Wave Equation

1. Verify that the following $u(x,t)$ satisfies the wave equation (1-D)
   a. $u = \sin at \sin bx$
2. Solve the one-dimensional wave equation with the following initial conditions
   $$u_{tt} = u_{xx}, u(0,t) = 0, u(1,t) = 0$$
   a. $u(x,0) = k\sin 3\pi x, u_t(x,0) = 0$
   b. $u(x,0) = \begin{cases} 2x(1-x) & x \in \left[0,\frac{1}{2}\right] \\ 0 & x \in \left[\frac{1}{2},1\right] \end{cases}, u_t(x,0) = 0$
3. Solve the following wave equation for $u(x,y,t)$
   $$u_{tt} = (u_{xx} + u_{yy})$$
   $$u(x,b,t) = u(0,y,t) = u(a,y,t) = u(x,0,t) = 0$$
   $$u(x,y,0) = \sin\frac{6\pi x}{a}\sin\frac{2\pi y}{b}, u_t(x,y,0) = 0, 0 \leq x \leq a, 0 \leq y \leq b$$

Heat Equation

1. Solve the Heat equation (1-D) for the following initial condition $(0 \leq x \leq L)$
   a. $u(x,0) = \cos 2x$
2. Verify that the following $u(x,t)$ satisfy the Heat equation (1-D)
   a. $u = e^{-\pi^2 t}cos25x$

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3. Obtain the solution of the heat equation in integral form for the following conditions

$$u_t = u_{xx}, u(0,t) = 0, u(1,t) = 0$$

   a. $u(x,0) = \begin{cases} 1 & |x| < 1 \\ 0 & Otherwise \end{cases}$
   b. $u(x,0) = \begin{cases} |x| & |x| < 1 \\ 0 & Otherwise \end{cases}$

Laplace and Poisson Equation

1. Verify that the potential $u = \frac{c}{r} + k, r = \sqrt{x^2 + y^2 + z^2}$ satisfies Laplace equation in spherical coordinates.
2. Verify that $u = c\ ln\, r + k, r = \sqrt{x^2 + y^2}$ is a solution of Laplace equation in cylindrical coordinates
3. Verify that the following $u(x, y)$ satisfies the Laplace equation (2-D)
   a. $u = \cos y \sinh x$
   b. $u = \frac{x}{x^2+y^2}$
4. Verify that the following $u(x, y)$ satisfy the Poisson equation (2-D) with $f(x, y)$
   a. $u = \sin xy, f = (x^2 + y^2) \sin xy$