INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI
DEPARTMENT OF MATHEMATICS AND STATISTICS

MA6024-PARTIAL DIFFERENTIAL EQUATIONS

Assignment-4

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1. Find the complete integral in each problem by Charpit's method. Here $p = u_x$ and $q = u_y$

   (a) $u = px + qy + pq$

   (b) $p^2 + px + q = u$

   (c) $(p + q)(u - px - qy) = 1$

   (d) $p + q = 3pq$

   (e) $p^2 x + q^2 y = u$

   (f) $pq = 1, u(x, 0) = \log x$

   (g) $pq = u, u(x, 1) = x, 0 \leq x \leq 1$

   (h) $pq = 2, u(x, x) = 3x, 0 \leq x \leq 1$

   (i) $p^2 + q^2 + 2(p - x)(q - y) - 2u = 0, u(x, 0) = 0, 0 \leq x \leq 1$

   (j) $p^3 - q = 0, u(x, 0) = 2x\sqrt{x}, 0 \leq x \leq 1$

   (k) $p + \frac{1}{2}q^2 = 1, u(0, y) = y^2, 0 \leq y \leq 1$

   (l) $u = xp + yq + \frac{p^2+q^2}{2}$

   (m) $2p^2 x + qy = u, u(x, 1) = \frac{1-x^2}{2}$

   (n) $p^2 + q^2 = 1, u(x, 0) = 0$

   (o) $p^2 + q^2 = u, u(x, 0) = x^2 + 1$

   (p) $p^2 + q^2 = u^2, u(x, 0) = 1$

   (q) $p^2 + q = 0, u(x, 0) = x$

   (r) $q^2 - p = 0, u(x, 0) = f(x)$

   (s) $p^2 + q^2 = 1, u(\cos s, \sin s) = 0$

   (t) $p^2 - 3q^2 = u, u(x, 0) = x^2$

2. Find a complete integral in each problem by Jacobi's method. Here $p = u_x, q = u_y$ and $r = u_z$,

   (a) $p^2 x + q^2 y = u$

(b) $2pxz + 3qz^2 + q^2r = 0$

(c) $(p^2 + q^2)y = qu$

(d) $px + qy - r^3 = 0$

(e) $u^2 = pqxy$

(f) $(y + z)(q + r)^2 + up = 0$

(g) $2(y + uq) = q(xp + yq)$

(h) $2pxz + 3qz^2 + q^2r = 0$

3. Find the envelope of the family of lines given by $y = mx \pm a\sqrt{1 + m^2}$

4. Find the envelope of

$$G(x, y, z, \lambda) = \lambda x + \sqrt{1 - \lambda^2 - \mu^2 y} + \mu z = 0$$

where μ is a constant and λ is a real parameter

5. Direction numbers of the normal to the tangent plane $(\cos\beta, \sqrt{1 - \cos^2\beta - \sin^2\alpha}, \sin\alpha)$. Find the envelope of the family of planes given by

$$(\cos\beta)x + (\sqrt{1 - \cos^2\beta - \sin^2\alpha})y + (\sin\alpha)z = 0$$

where α is a constant and β is a real parameter.

6. Show that

$$S = \sqrt{\alpha}x + \sqrt{1 - \alpha}y + \beta, \alpha, \beta \in \mathbb{R}, 0 < \alpha < 1$$

is a complete integral of $S_x - \sqrt{1 - S_y^2} = 0$. Find the envelope of this family of solutions.

7. Is an envelope of a family of planes always a cone? Justify

8. Solve the following Riemann problem. Let $\Omega = \{(x, y) \in \mathbb{R}^2 : x > y\}$

$$u_x + u_y = 0$$

$$u(x, 0) = \begin{cases} u_l & x < 0 \\ u_r & x > 0 \end{cases}$$

with $u_l \neq u_r$

9. Solve the following Riemann problem. Let $\Omega = \{(x, y) \in \mathbb{R}^2 : x < y\}$

$$u_x + u_y = 0$$

$$u(x, 0) = \begin{cases} u_l & x < 0 \\ u_r & x > 0 \end{cases}$$

with $u_l \neq u_r$

Bonus: 5 Marks

Write a short note (each topic, at least one page) on Rankine-Hugonoit Jump condition, Entropy Condition and Olinik's Condition with an example problem.