



INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI  
DEPARTMENT OF MATHEMATICS AND STATISTICS

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MA6024-PARTIAL DIFFERENTIAL EQUATIONS

Assignment-5

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1. Reduce the equation following equation to canonical form

(a)  $u_{xx} + x^2u_{yy} = 0$

(b)  $2u_{xx} - 4u_{xy} + 2u_{yy} + 3u = 0$

(c)  $u_{xx} + yu_{yy} = 0$

(d)  $u_{xy} + u_x + u_y = 2x$

(e)  $u_{xx} - 6u_{xy} + 12u_{yy} + 4u_x - u = \sin(xy)$

2. Classify the following PDEs as hyperbolic, parabolic and elliptic and obtain their region (wherever applicable). Obtain their characteristics, reduce it to canonical form and then find the solution

(a)  $(1 - M_\infty^2)\phi_{xx} + \phi_{yy} = 0$

(b)  $u_{xx} + x^2u_{yy} = 0$

(c)  $u_{xx} - xu_{yy} = 0$

(d)  $\left(1 - \frac{u^2}{c^2}\right)\phi_{xx} - 2\frac{uv}{c^2}\phi_{xy} - \left(1 - \frac{v^2}{c^2}\right)\phi_{yy} = 0$

3. How do you classify second-order PDEs in  $n$ - variables as hyperbolic, parabolic and elliptic?

4. Give at least five examples for second-order elliptic PDEs in 3-variables

5. Give at least five examples for second-order parabolic PDEs in 3-variables

6. Give at least five examples for second-order hyperbolic PDEs in 3-variables

7. How do you classify first-order system of PDEs as elliptic, hyperbolic, parabolic, elliptic-hyperbolic?

8. Give an example for a first-order elliptic PDE system

9. Give an example for a first-order parabolic PDE system

10. Give an example for a first-order hyperbolic PDE system

11. Give an example for a first-order elliptic-hyperbolic PDE system

## Bonus: 5 Marks

Classify the following Euler equations for unsteady one-dimensional flow, obtain their characteristics and general solution

$$\begin{aligned}\rho_t + u\rho_x + \rho u_x &= 0 \\ u_t + uu_x &= -\frac{c^2}{\rho}\rho_x\end{aligned}$$

where  $c$  is the speed of sound and the relation between the pressure and density is given by

$$c^2 = \frac{\partial p}{\partial \rho}$$

Hint: You can Refer the following link for system of PDEs (or you can do google search)  
[https://www.iist.ac.in/sites/default/files/people/IN08026/Canonical\\_form.pdf](https://www.iist.ac.in/sites/default/files/people/IN08026/Canonical_form.pdf)