भारतीय प्रौद्योगिकी संस्थान तिरुपति
TIRUPATI

Tutorial Sheet for PDE: MA 5023 - 16-Decmber-2020

Indian Institute of Technology Tirupati

Department of Mathematics and Statistics

MA5023 – Differential Equations for Engineers –Tutorial Sheet-PDE

Wave Equation

1. Verify that the following $u(x,t)$ satisfies the wave equation (1-D)
   a. $u = x^2 + t^2$
   b. $u = \cos 4t \sin 2x$
2. Solve the one-dimensional wave equation with the following initial conditions
$$u_{tt} = u_{xx}, u(0,t) = 0, u(\pi,t) = 0$$
$$u(x,0) = 0, u_t(x,0) = \begin{cases} 0.01x & x \in \left[0, \frac{\pi}{2}\right] \\ 0.01(\pi - x) & x \in \left[\frac{\pi}{2}, \pi\right] \end{cases}$$
3. Solve the one-dimensional wave equation with the following initial conditions
$$u_{tt} = u_{xx}, u(0,t) = 0, u(1,t) = 0$$
   a. $u(x,0) = kx(1-x), u_t(x,0) = 0$
   b. $u(x,0) = kx^2(1-x), u_t(x,0) = 0$

Heat Equation

4. Solve the Heat equation (1-D) for the following initial condition $(0 \le x \le L)$
   a. $u(x,0) = x,$
   b. $u(x,0) = 1,$
5. Verify that the following $u(x,t)$ satisfy the Heat equation (1-D)
   a. $u = e^{-t} \sin x$
   b. $u = e^{-\omega^2 c^2 t} cos\omega x$
6. Obtain the solution of the heat equation in integral form for the following conditions
$$u_t = u_{xx}, u(0,t) = 0, u(1,t) = 0$$
   a. $u(x,0) = \frac{1}{1+x^2}$
   b. $u(x,0) = \frac{\sin x}{x}$

Laplace Equation

7. Verify that the following $u(x,y)$ satisfies the Laplace equation (2-D)
   a. $u = e^x cosy$
   b. $u = \tan^{-1}\frac{y}{x}$

Poisson Equation

8. Verify that the following $u(x,y)$ satisfy the Poisson equation (2-D) with $f(x,y)$
   a. $u = \frac{y}{x}, f = \frac{2y}{x^3}$