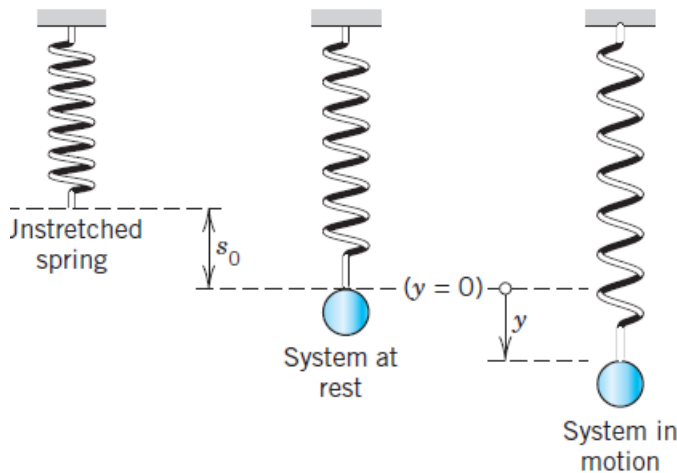
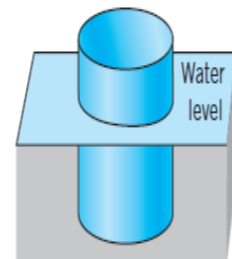


Second Order ODE, Variations + BVP

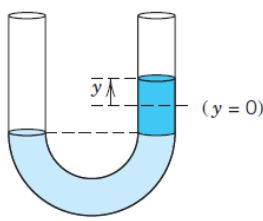


Problem 1: An ordinary coil spring resists extension as well as compression. We attach an iron ball at its lower end. When the system is at rest after attaching the iron ball, we say it as the initial position. When we pull the ball down, the system experiences a force. What will be the undamped system?

Problem 2 (a) According to Archimedean principle buoyance force equals the weight of the water displaced by the body. A cylindrical buoy of diameter 60cm is floating in water with its axis vertical. When depressed downward in the water and released, it vibrates with period 2 sec. What is its weight?



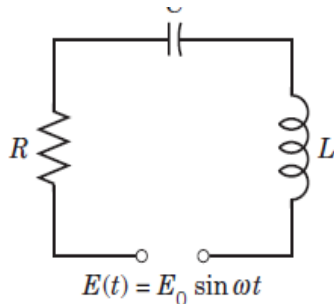
$$y'' + \omega^2 y = 0$$



(b) What is the frequency of vibration of 5 litres of water in a U-Shaped tube of diameter 4cm? (Neglect friction).

$$y'' + \omega^2 y = 0$$

- (i) $m = 10, k = 90$ (ii) $m = 1, k = 3$



Problem 3: Model the following RLC-Circuit and solve for I

$$LI' + RI + \frac{I}{C} = E(t)$$

$$I = Q' \Rightarrow LQ'' + RQ' + \frac{Q}{C} = E(t)$$

Upon differentiation I w.r.to t , $LI'' + RI' + \frac{I}{c} = E'(t)$

- (a) $R = 24 \Omega, L = 1.2 H, C = \frac{1}{90} F, E = 220 \sin 5 t V$

Problem 4: Reduce to First-order and Solve

- (a) $yy'' = 3y'^2$
(b) $x^2y'' + xy' - 4y = 0, y_1 = x^2$

Problem 5: Solve the following

- (a) $y'' + 2k^2y' + k^4y = 0$
(b) $10y'' - 32y' + 25.6y = 0$
(c) $4y'' - 4y' - 3y = 0$

Problem 6: Find the Wronskian and show linear independence

- (a) $2x, \frac{1}{4x}$ (b) x^3, x^2 (c) $e^{-x}\cos\omega x, e^{-x}\sin\omega x$

Problem 7: Find a second-order homogeneous linear ODE for which the given functions are solutions. Show linear independence by Wronskian. Solve the IVP

- (a) $\cos 5x, \sin 5x, y(0) = 3, y'(0) = -5$

Problem 8: Solve the following non-homogeneous linear ODE

- (a) $y'' + 5y' + 6y = 2e^{-x}$
(b) $10y'' + 50y' + 57.6y = \cos x$

Problem 9: Solve by using the variation of Parameters

- (a) $y'' + 4y = \cos 2x$