भारतीय प्रौद्योगिकी संस्थान तिरुपति

TIRUPATI

Tutorial Sheet: MA5023 – 06, November 2020

Indian Institute of Technology Tirupati

Department of Mathematics and Statistics

MA5023 – Differential Equations for Engineers – Tutorial Sheet

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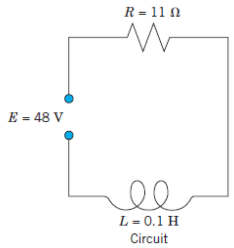
Mathematical Modelling

Problem 1: There was a murder in a hotel in room number 315 at 4:30 PM. Police arrested Arjun, who was in the next room at 5:00 PM. But, Arjun claims that he was not in his room for at least half an hour. The police checked the water temperature of his tea kettle in his room at the instant of arrest and again 30 minutes later, obtaining the values 87℃ and 43℃, respectively. Can you investigate the case as an inspector? Is it possible to claim that Arjun is the murderer? To answer this question, develop a model.

Problem 2: In a hostel, there is a cylindrical water tank of diameter 2m and height 2.25m. On a fine day, when Ragu was the first person to take the shower at 7 AM in the hostel, the tank was empty. After an inspection by the hostel warden, the warden found a circular hole in the water tank. When the hostel watchman switched off the power button of the water tank at 1 AM on the day, it was completely filled. Without manually measuring the diameter of the hole, could you calculate the diameter of the hole? To answer this question, develop a model.

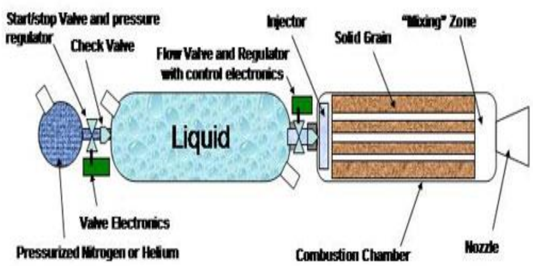
Problem 3: One hour before surgery, a certain drug at a constant amount was injected to the patient's bloodstream. A certain amount of drug is removed simultaneously to avoid overdosage of drugs which is proportional to the amount of the drug present at time *t*. Model the problem.

Problem 4: Model the following RL-Circuit for the current under the assumption that the initial current is zero.



(Hint: Ohm's Law: $V = I\,R$, **Kirchhoff's Voltage Law:** $Voltage\ drop + V = E, LI' + IR = E$)

$$R = 11\ \Omega, L = 0.1\ H, E = 110\sin 10\ t\ V$$



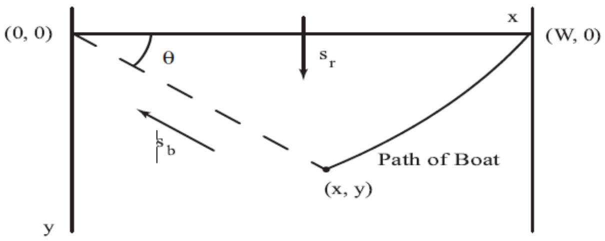
Problem 5: A hybrid fuel tank in a rocket works on the principle of mixing two different fuel substance for combustion which in turn produces fuel supply for the throttle. The first tank contains 2 million litres of fuel in which another solid fuel substance of 0.18 million kg is dissolved. Each 50 litre of the fuel fed into the throttle after mixing contains (1 + *cos t*) kg of the dissolved solid fuel substance per litre. The mixture is uniform and runs to the throttle at the same rate. Model amount of solid fuel substance at any time *t*.

Problem 6: In the city of Hamelin, the rat population was a big problem. It was initially assumed that the rate of change of rat population w.r.to time is equal to twice its population at any given day. Until the pied piper arrived, people killed rats and hence the growth rate of the rat population decreased in proportion to the population every day. Model the problem to answer “How many years will be required to have a rat-free city if the initial and 1-year population of the rat are respectively 2 million and 1.5 million?”

Problem 7: A mummy in Egypt experimented with radiocarbon dating to find its age. Approximately, when did the mummy die, if the ratio of carbon ${}^{14}_{6}C$ to carbon ${}^{12}_{6}C$ in this mummy is 52.5% of that of a living organism?

Problem 8: The efficiency of the engines of subsonic aeroplanes depends on air pressure and is usually maximum near 10668 metres. The rate of change in air pressure is proportional to the pressure. At 5486.4 metre height, the air pressure is half of its value at the sea level. Model the problem.

Problem 9: The Gompertz model is $y' = -Ay\ln y, A > 0$, where $y(t)$ is the mass of tumour cells at time t. The declining growth rate with increasing $y > 1$ corresponds to the fact that cells in the interior of a tumour may die because of insufficient oxygen and nutrients. Model the problem.



Problem 10: Path of a Boat in a river: The y −axis and the line $x = W > 0$ represent the banks of a river. The river flows in the negative y −direction with speed s_r. A boat whose speed in still water is s_b is launched from the point $(W, 0)$. The boat is stirred so that it is always headed toward the origin. The components of the boat velocity in x- and y-direction are $\frac{dx}{dt} = -s_b cos\theta, \frac{dy}{dt} = -s_r + s_b sin\theta$. Solve for dy/dx, if $W = 0.5km, s_r = 3kmph, s_b = 3kmph$.