भारतीय प्रौद्योगिकी संस्थान तिरुपति
TIRUPATI

Tutorial Sheet: MA 5023 - 7-December-2020

Indian Institute of Technology Tirupati

Department of Mathematics and Statistics

MA5023 – Differential Equations for Engineers – Tutorial Sheet

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Problem 1: Legendre's equation

(a) Using $n = 0$, prove that $P_0(x) = 1$ and $y_2(x) = x + \frac{1}{3!}x^3 + \frac{1}{5}x^5 + \cdots = \frac{1}{2}\ln\frac{1+x}{1-x}$
Verify this by solving, the Legendre equation with $n = 0$.

(b) Using $n = 1$, prove that $y_2(x) = P_1(x) = x$ and

$$y_1 = 1 - x^2 - \frac{1}{3}x^4 - \frac{1}{5}x^6 - \cdots = 1 - \frac{1}{2}x\ln\frac{1+x}{1-x}$$

Problem 2: Legendre's equation

Applying the binomial theorem to $(x^2-1)^n$, differentiating it n times term by term, and comparing the result with $P_n(x)$, show that

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n[(x^2-1)^n]}{dx^n}$$

Problem 3: Bessel's equation: Prove the following

$$(a) J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x, \qquad (b)\ J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x, (c)\ J_{-n}(x) = (-1)^n J_n(x),$$

Problem 4: Find general solution in terms of J_ν, Y_ν

a) $x^2y'' + xy' + (x^2 - 16)y = 0$
b) $xy'' - 5y' + xy = 0, y = x^3u$

Fourier Series and Fourier Integral

Problem 5: Find the Fourier coefficients of the periodic function $f(x)$

$$a)\ f(x) = \begin{cases} -k & if \quad -\pi < x < 0 \\ k & if \quad 0 < x < \pi \end{cases} \ and\ f(x+2\pi) = f(x)$$

$$b)\ f(x) = \begin{cases} x & if \quad -\pi < x < 0 \\ \pi - x & if \quad 0 < x < \pi \end{cases} \ and\ f(x+2\pi) = f(x)$$

Problem 6: Find the Fourier sine integral representation of the function

$$a)\ f(x) = \begin{cases} x & if \quad 0 < x < 1 \\ 0 & if \quad x > 1 \end{cases}$$

Problem 7: Prove that $y_m(x) = \sin mx, m = 1,2, \ldots$ form an orthogonal set on $[-\pi, \pi]$.