भारतीय प्रौद्योगिकी संस्थान तिरुपति
TIRUPATI

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**Indian Institute of Technology Tirupati**

**Department of Mathematics and Statistics**

**MA5023 – Differential Equations for Engineers – Tutorial Sheet**

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## First Order ODEs

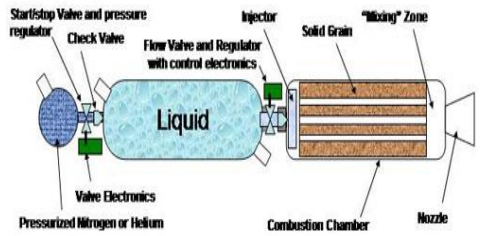
**Problem 1:** Solve the following first-order ODEs

(a) $\frac{dT}{dt} + kT = kT_m$ (b) $\frac{dv}{dt} + \frac{k}{m}v = g$

(c) $\frac{dQ}{dt} + \left(\frac{f}{V_0+et-ft}\right)Q = be$ (d) $\frac{dq}{dt} + \frac{1}{RC}q = \frac{E}{R}$

**Problem 2:** A bacteria culture is known to grow at a rate proportional to the amount present. After one hour, 1000 strands of the bacteria are observed in the culture; and after four hours, 3000 strands. Find the approximate number of strands of the bacteria originally in the culture.

**Problem 3:** A hybrid fuel tank in a rocket works on the principle of mixing two different fuel




substance for combustion which in turn produces fuel supply for the throttle. The first tank contains 2 million litres of fuel in which another solid fuel substance of 0.18 million kg is dissolved. Each 50 litre of the fuel fed into the throttle after mixing contains $(1 + \cos t)$ kg of the dissolved solid fuel substance. The mixture is uniform and runs to the throttle at the same rate. What is the amount of solid fuel substance at any time $t$?

**Problem 4:** The Gompertz model is $y' = -Ay \ln y, A > 0$, where $y(t)$ is the mass of tumour cells at time $t$. The declining growth rate with increasing $y > 1$ corresponds to the fact that cells in the interior of a tumour may die because of insufficient oxygen and nutrients. Model and solve it.

**Problem 5: Hanging Cable:**
It can be shown that the curve $y(x)$ of an inextensible flexible homogeneous cable hanging between two fixed points is obtained by solving $y'' = k\sqrt{1 + y'^2}$, where the constant $k$ depends on the weight. This curve is called a catenary. Find $y(x)$, if $k = 1$ and the curve passes through the fixed points $(-1,0)$ and $(1,0)$ in a vertical $xy$-plane.

**Problem 6:** On what interval does each of the following initial value problems have a unique solution?

$$y' = \frac{y}{(x-1)(x+2)} + \frac{1}{x}, y(-1) = 2$$

**Problem 7: Solve the following IVP**

$y' + 2y = 4\cos 2x, y\left(\frac{\pi}{4}\right) = 3$ (b) $y' + y = y^2, y(0) = -\frac{1}{3}$