INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI
DEPARTMENT OF MATHEMATICS AND STATISTICS

MA6204-NUMERICAL ANALYSIS

Assignment-1

II MSc (Mathematics and Statistics), IV Semester

Last Date To Submit: **15 February 2021**

Each question carries 5 Marks

Little o and Big O

1. Prove or disprove the following:

   (a)
   $$\sum_{k=0}^{n} x^k = \frac{1}{1-x} + o(x^n) \quad \text{as} \quad x \to 0$$

   (b)
   $$\sum_{k=0}^{n} a_k x^k = O(x^n), x \geq 1$$

   (c)
   $$\cos x - 1 + \frac{x^2}{2} = O(x^k) \quad \text{as} \quad x \to 0$$

   (d)
   $$\ln x = o\left(\frac{1}{x^r}\right) \quad \text{as} \quad x \to 0, x \in (0,\infty), r > 0$$

   (e)
   $$\ln x = o(x^r) \quad \text{as} \quad x \to \infty, x \in (0,\infty), r > 0$$

   (f)
   $$3\log_8 n + \log_2(\log_2(\log_2 n)) = O(\log n)$$

   (g)
   $$0.3n + 5n^{1.5} + 2.5n^{1.75} = O(n^{1.75})$$

   (h)
   $$3n^2 + 10n\log n = O(n\log n)$$

   (i)
   $$10\sqrt{n} + \log n = O(n)$$

(j)

$$n^3 + 20n + 1 = O(n^3)$$

(k)

$$n^2 + 42n + 7 = O(n^2)$$

(l)

$$n^3 + 20n + 1 = \Omega(n^2)$$

2. Find real numbers, a, b and c, such that

$$\ln\left(\frac{\sin x}{x}\right) = ax^2 + bx^4 + cx^6 + o(x^8) \quad \text{as} \quad x \to 0, x \in (-\pi, \pi)$$

3. Prove that for any constants $a, b, c > 0$,

$$O(a) \subset O(\log n) \subset O(n^b) \subset O(c^n)$$

4. Prove that for any $a < b$.

$$O(n^a) \subset O(n^b)$$

5. Verify whether the following are true or not.

(a)

$$\frac{5}{n} + e^{-n} = O\left(\frac{1}{n}\right)$$

(b)

$$e^{-n} = o\left(\frac{1}{n^2}\right)$$

(c)

$$\ln 2 - \sum_{k=1}^{n-1} (-1)^{k-1}\frac{1}{k} = O\left(\frac{1}{n}\right)$$

(d)

$$e^x - \sum_{k=0}^{n-1} x^k \frac{1}{k!} = O\left(\frac{1}{n!}\right) \qquad (|x| \leq 1)$$

6. Prove that

$$10\ln(n) + 5(\ln(n))^3 + 7n + 3n^2 + 6n^3 = O(n^3)$$

7. Prove that $\frac{1}{2^n}$ converges linearly with rate of convergence $\frac{1}{2}$

8. Prove that $\frac{1}{2^{2^n}}$ converges superlinearly or quadratically

9. Prove that $\frac{1}{2^{n^2}}$ converges superlinearly.

10. Prove that $\frac{1}{n+1}$ converges logarithmically or sublinearly.

11. Prove that $\frac{1}{n^k}, k > 0$ converges linearly.

12. Prove that

$$\frac{n^\alpha}{n+1} \to \begin{cases} 0 & \text{if } \alpha < 1 \\ 1 & \text{if } \alpha = 1 \\ \infty & \text{if } \alpha > 1 \end{cases}$$

Numerical Errors

13. Suppose that you have the task of measuring the lengths of a bridge and a rivet and come up with 9999 and 9 cm, respectively. If the true values are 10,000 and 10 cm, respectively, compute (a) E_t, (b) E_{tabs} and (c) ϵ_t for each case.

14. Evaluate e^{-5} using two approaches

$$e^{-x} = \sum_{k=0}^{\infty} \frac{(-x)^k}{k!}$$

and

$$e^{-x} = \frac{1}{e^x} = 1/\sum_{k=0}^{\infty} \frac{x^k}{k!}$$

and compare with true value of 6.737947×10^{-3}. Use first 20 terms to evaluate each series. Compute E_{tabs} and ϵ_t. Write a C++ or Python code to compute the first 20 terms.

15. Evaluate the polynomial

$$y = x^3 - 5x^2 + 6x + 0.55$$

at $x = 1.37$. Use 3-digit arithmetic with chopping. Compute ϵ_t

16. Evaluate the polynomial

$$y = x(x(x-5) + 6) + 0.55$$

at $x = 1.37$. Use 3-digit arithmetic with roundoff. Compute ϵ_t